

ρ Parameter Constraints on Models with Large Compact Dimensions

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Abstract

In models with large extra dimensions, where quantum gravity effects become strong at the TeV scale, the ρ parameter can receive large contributions from one-loop diagrams involving exchange of multiple graviton and dilaton states. These contributions are computed, taking into account cancellation of spurious infrared divergences, and the (finite) results for $d = 5$ and 6 are compared with current experimental data. It is shown that 5 large extra dimensions are incompatible with the data and $d = 6$ is severely constrained.

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1. Introduction

Considerable excitement has been generated by the suggestion [1] that strong gravitational effects could become manifest at scales of the order of a few TeV. One of the ways in which this can happen requires large compactified spacetime dimensions beyond the four Minkowski dimensions. An elegant model involves a string theory [2] (living in 10 dimensions) which has solitonic excitations of the gravitational field called D -branes [3]. In this model, proposed [4] by Antoniadis, Arkani-Hamed, Dimopoulos and Dvali (AADD), all Standard Model (SM) fields are conceived of as living on a heavy D_3 -brane embedded in a space of 10 dimensions. In fact, the SM fields correspond to excitations of open strings whose ends are confined to the brane. Of the extra 6 dimensions, it is possible to have d of them compactified with radii R_c , where R_c can be large, while the remaining $(6-d)$ dimensions are compactified with radii near the Planck length $M_{Pl}^{-1} \sim 10^{-33}$ cm. The latter do not play much part in the subsequent discussion, though their existence is essential to have a consistent string theory in the first place. For all practical purposes, therefore, spacetime consists of $(4+d)$ dimensions, the extra (spatial) d dimensions being compactified, typically on a torus $T^{(d)}$ with radius R_c each way. This is called the *bulk*, as opposed to the *brane* on which the observable Universe lives. Since gravity experiments have not really probed the sub-millimetre

The actual value of Newton's constant $G_N^{(4+d)}$ in the bulk can be taken as large as $\mathcal{O}(1 \text{ TeV}^{-2})$, but its value $G_N^{(4)}$ in the effective 4-dimensional space at length scales $\gg R_c$ is the extremely small one $\mathcal{O}(10^{-32}) \text{ TeV}^{-2}$ measured in gravity experiments. The two are related, using Gauss' Law [6], by

$$\left[M_{Pl}^{(4)} \right]^2 \sim R_c^d \left[M_{Pl}^{(4+d)} \right]^{2+d}$$

where $M_{Pl}^{(4+d)} \simeq \left[G_N^{(4+d)} \right]^{-\frac{1}{2+d}}$ denotes the Planck mass in the relevant number of dimensions. If $M_{Pl}^{(4+d)} \sim 1 \text{ TeV}$, then $R_c \sim 10^{30/d-19} \text{ m}$, *i.e.*, for $d = 1$, $R_c \sim 10^{11} \text{ m}$, which predicts deviations from Einstein gravity at solar system scales. Since no such effects

are seen, we are constrained³ to take $d \geq 2$. For these values $R_c < 1$ mm and there is no conflict with known facts[5]. The smallness of Newton’s constant $G_N^{(4)}$ is thus a direct consequence of the compactification and hence there is no hierarchy problem in this theory⁴.

In traditional Kaluza-Klein (KK) theories [8], the mass-spectrum of non-zero KK modes is driven to the Planck scale $M_{Pl}^{(4)}$. This problem is avoided in the AADD model by having the SM particles live on a ‘wall’ with negligible width (which we identify with the D_3 brane). While SM fields are confined to the brane, gravitons correspond to excitations of closed strings propagating in the bulk. The only new effects observable on the brane — to which corresponds the observable Universe — will be those due to exchange of gravitons between SM particles on the brane.

To construct an effective theory in 4 dimensions, *i.e.* on the brane, gravity may be quantized taking the usual weak-field (‘linearized’) limit, assuming that the underlying string theory will ultimately take care of the well-known ultraviolet problems. Interactions of gravitons now follow from the $(4+d)$ -dimensional Einstein equations in the compactification limit. Feynman rules to the lowest order in $\kappa = \sqrt{16\pi G_N^{(4)}}$ for this effective theory have been worked out in detail in Refs. [9] and [10]. We make use of the prescriptions of Han, Lykken and Zhang [10] in our work. For convenience, some of the relevant Feynman rules are listed in Appendix A.

In this effective theory, the couplings of the gravitons to the SM particles will be suppressed by the Planck scale $M_{Pl}^{(4)} \simeq 1.2 \times 10^{19}$ GeV. This is offset, however, by the fact that, after compactification, the density of massive KK graviton states is very high, being indeed, proportional to $[M_{Pl}^{(4)}]^2$. The $M_{Pl}^{(4)}$ dependence cancels out, therefore, leaving an interaction of electroweak strength, whose scale is set by the bulk Planck’s constant $M_S \sim M_{Pl}^{(4+d)} \sim 1$ TeV, henceforth called the ‘string’ scale. A further assumption usually

³Of course, for $d = 1$, we have $R_c \propto M_S^{-3}$: thus $M_S > 10^4$ TeV would make $d = 1$ viable. However, then there are no interesting collider signals.

⁴A related problem, that of stabilization of the compactification scale, exists, however; this has been discussed in Ref. [7]. In fact, this is the chief criticism of the AADD model.

made is that the brane itself is heavy and its vibration modes decouple from the processes under consideration. This corresponds to a static approximation for the brane [11].

The interactions generated by gravity between SM particles on the brane can be written mostly in terms of exchange of spin-0 and spin-2 gauge bosons. At the lowest order in κ , the spin-1 gauge bosons decouple [6, 11] from matter due to the diagonal structure of the energy-momentum tensor for matter fields on the brane. Interactions of the spin-0 dilaton (or ‘radion’, R) and the spin-2 gravitons (G) — or rather, their massive KK modes — have been described in Refs. [9] and [10] and several phenomenological studies *at the tree-level* have been made using these interactions. The salient features of such studies are briefly surveyed in the next section. However, it suffices to note that almost all of these tend to provide *lower* bounds on the string scale M_S .

One-loop effects of the interactions of towers of KK states of the graviton and radion have not yet been investigated in detail. The pioneering work in this direction was in section 3.5 of Ref. [10], in which a leading-order calculation of self-energy corrections to the mass of a scalar particle were worked out. A more detailed calculation of the anomalous magnetic moment of the muon was attempted in Ref. [12]. In the latter, the result turned out to be ‘remarkably finite’, though the actual numbers were rather disappointingly small. It was argued in Ref. [12] that one-loop effects are worth calculating even though there may be other TeV-suppressed operators at the tree-level, simply because the one-loop effects are completely calculable, given the Feynman rules. One can add the argument that a study of one-loop effects brings out subtle features of the theory in a way that tree-level effects can never be expected to do.

In the present work, therefore, we compute one-loop corrections to the masses of the electroweak gauge bosons W and Z , which are constrained by the famous ρ parameter [13]. Extra contributions to the ρ parameter have been discussed in the context of AADD-type models in the literature [14], but not in the context of one-loop calculations. It turns out that the one-loop contributions from virtual radion and graviton states are strongly divergent in the ultraviolet (UV), a feature which is to be expected in an effective theory.

The use of the string scale M_S as a momentum cutoff has been suggested in Ref. [10] (and elsewhere) and this leads to large contributions which actually *grow* with increasing M_S . Recognising that this is not a pathological feature but simply a manifestation of the fact that we have an effective (non-renormalisable) theory, we can therefore, use the experimental bounds on the ρ parameter to derive upper bounds on this scale M_S , unlike all the tree-level processes considered hitherto, which are suppressed by powers of M_S .

An interesting feature which emerges in our calculation is the presence of infrared (IR) divergences (for $d = 2, 3$ and 4) in the summation over masses of particles in the KK towers. These are due to the zero modes of the graviton (and radion) KK states and arise ultimately from the fact that in the IR limit when the graviton momenta are comparable to the mass gap between neighbouring excitations, the continuum approximation is invalid. One is therefore led to believe that the IR divergences are spurious and should cancel out of a complete calculation⁵ A useful suggestion made by the authors of Ref. [10] is to regulate these expressions by simply dropping the zero mode (whose coupling is Planck-scale suppressed) and start from the first massive KK mode. Cancellation of the IR divergence will therefore manifest itself as a cancellation of the dependence of the final result on the mass of this mode. We shall see that with the present level of knowledge, the presence of these IR divergences makes the ρ -parameter calculation non-predictive for $d = 2, 3$ and 4. However, these same IR divergences allow us to make a concrete prediction for $d = 5$ and 6, even without a complete knowledge of the relevant Feynman rules.

The plan of this article is as follows. Section 2 consists of a micro-review of phenomenological studies of the AADD model carried out at the tree level. In the next section, we describe the basic formalism required to calculate the W, Z self-energy corrections (and hence the ρ parameter) in the AADD model. Section 4 is devoted to numerical results from the calculation and their interpretation. It is in this section that we discuss

⁵ One should also note that gravity is known to be infrared-safe, especially in the full $(4+d)$ -dimensional theory. Compactification corresponds to replacing certain components of momentum by masses and this cannot generate IR divergences. The authors are grateful to Ashoke Sen for bringing this fact to their notice.

the cancellation of IR divergences. We summarize our results in Section 5. Appendix A comprises of the relevant Feynman rules. The machinery of one-loop integrals using the ultraviolet (UV) cutoff M_S is described in detail in Appendix B.

2. Phenomenological Studies

Since gravitons (and radions) couple to any particle with a non-vanishing energy-momentum tensor, it is possible to make a variety of phenomenological studies of the interactions of their KK modes. Though these have not yet been completely explored, several important results are already available. These investigations can be classified into two types: those involving *real* KK graviton production, and those involving *virtual* graviton exchange. A real KK mode of the graviton will have individual interactions with matter suppressed by $M_{Pl}^{(4)}$ and will, therefore, escape any detector of terrestrial proportions. One can, therefore, see signals [15] with large missing momentum and energy if an observable particle is produced in association with a tower of KK gravitons. Cross-sections for these depend explicitly on d , the number of extra dimensions. Each process can be used to obtain a bound on the string scale M_S for a given d . The most dramatic of these is $M_S > 50$ TeV for $d = 2$ and it comes from a study [16] of the flux of neutrinos from the supernova SN1987A. However, this astrophysical bound drops to about a TeV as soon as we go to $d > 3$. Most of the other processes lead to lower bounds of about 1–1.1 TeV on the string scale for $d = 2$, and are even weaker for $d > 3$.

Virtual (KK) graviton exchanges lead to extra contributions to processes involving SM particles in the final state and can be observed as deviations in the cross-sections and distributions. After summing over all the KK modes of the graviton, a tree-level Feynman amplitude involving graviton exchange is proportional to λ/M_S^4 , where

$$\begin{aligned} |\lambda| &\simeq \log(M_S^2/s) \quad \text{for } d = 2, \\ &\simeq (d-2)^{-1} \quad \text{for } d > 2, \end{aligned} \tag{1}$$

to the leading order in s/M_S^2 , where \sqrt{s} is the scale of the interaction [9, 10]. It has been found convenient to absorb the magnitude of λ into M_S^{-4} , so that bounds from virtual processes can be presented in a compact form. However, it should be borne in mind that the effective string scale M_S is, then, $\widetilde{M}_S = \lambda^{-1/4} M_S$, and hence different from the actual M_S , such as is considered for real graviton emission. Once this slight abuse of notation is understood, each process can be used [17] to obtain a bound on \widetilde{M}_S for a given sign of λ . Among the most stringent of these bounds are $\widetilde{M}_S > 1$ TeV for $\lambda = \pm 1$ which come from studies of experimental data on dileptons [18] and dijets [19] at the Fermilab Tevatron.

A summary of the most stringent lower bounds from collider data on M_S , as well as the astrophysical bound from a study of the neutrino flux from the supernova SN1987A, is given in Table 1.

Process	\widetilde{M}_S	$d = 2$	$d = 3$	$d = 4$	$d = 5$	$d = 6$
$p\bar{p} \rightarrow \ell^+ \ell^-$	~ 1.0	~ 1.2	1.0	0.84	0.76	0.71
$p\bar{p} \rightarrow$ dijets	~ 1.1	~ 1.3	1.1	0.92	0.84	0.78
astrophysical bound		~ 50	~ 4	~ 1		

Table 1. Illustrating lower bounds on the string scale M_S (in TeV) from dilepton [18] and dijet [19] data at the Fermilab Tevatron. The astrophysical bounds from a consideration [16] of the neutrino flux at the supernova SN1987A are also shown for purposes of comparison.

4. Calculation of Self-Energies

In electroweak theory, the ρ parameter is defined by [13]

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} \quad (2)$$

to all orders in perturbation theory. The most convenient formalism for the study of this parameter is that given by Peskin and Takeuchi [20], who define $\rho - 1 = \alpha T$, where α is the fine structure constant and ρ is calculated in the zero momentum limit. The T -parameter defined above is one of the so-called *oblique* parameters and is zero in the Standard Model at tree-level. Non-vanishing values of T measure, therefore, one-loop corrections to the

W, Z masses in the SM and/or any new physics effects in these masses. The current experimental bounds on the (dimensionless) T -parameter are $T = -0.21 \pm 0.16 (+0.10)$.

In terms of the self-energy corrections $\Pi(p^2)$ of the gauge bosons W^\pm and Z^0 , the T -parameter is [20]

$$T = \frac{4\pi}{M_W^2} \left[\Pi_{WW}(0) - \Pi_{ZZ}(0) \cos^2 \theta_W \right] \quad (3)$$

where p is the propagator momentum and the self-energies are calculated in the limit $p^2 \rightarrow 0$. This will have small contributions in the Standard Model. The excess contribution to the T -parameter in the AADD model (henceforth denoted simply δT) can be calculated simply by evaluating the self-energy corrections of the W and Z bosons due to graviton (radion) loops.

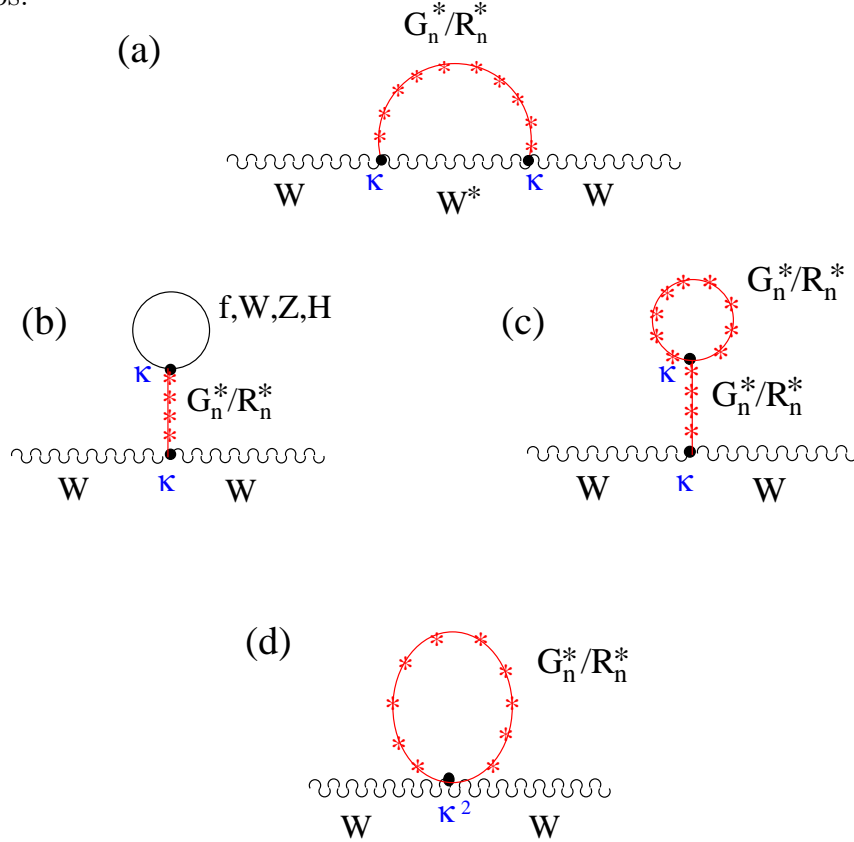


Figure 1. *Feynman diagrams corresponding to self-energy corrections of the W -boson in the AADD model at the one-loop level. G_n and R_n correspond to graviton excitations with spin 2 and 0 respectively. There will be a similar set of diagrams for the Z -boson.*

The Feynman diagrams corresponding to the self-energy of the W -boson are shown in

Figure 1(a-d). Since the low energy theory is just linearized Einstein gravity, we expect it to be non-renormalizable. Hence, rather than work with a whole family of effective operators ⁶, we work with the unrenormalized vertices as given in Ref. [10] and it is therefore necessary to include tadpole graphs such as those of Figure 1(b) and 1(c). These can have all kinds of particles in the loops, including (b) all SM particles and (c) the quanta of linearised gravity. Moreover (as pointed out in Ref. [10]) it is necessary to include ‘seagull’-type diagrams such as that in Figure 1(d). It is important to note that while the diagrams in Figure 1(a-c) involve $\mathcal{O}(\kappa)$ vertices, the diagrams in Figure 1(d) involve $\mathcal{O}(\kappa^2)$ vertices. Unfortunately, Feynman rules are not readily available in the AADD model for (i) the self-couplings in the pure gravity sector, which are required for the evaluation of Figure 1(c) and (ii) the full set of $\mathcal{O}(\kappa^2)$ vertices. Thus, any calculation of the self-energies at this stage must be incomplete to this extent. However, as we shall show, it is still possible to obtain some meaningful results, and this is the theme of this work.

Evaluation of the diagrams in Figure 1(a) and (b), using the Feynman rules given in Appendix A, is a long and tedious process. Some of the formalism is developed in Appendix B. The final result for $\Pi_{WW}^{AADD}(0)$ takes the form

$$\Pi_{WW}^{AADD}(0) \simeq \frac{M_S^2}{720\pi} \frac{1}{(2\sqrt{\pi})^d} \frac{1}{\Gamma(d/2)} \int_{x_0}^1 dx x^{-1+d/2} \left[J_W^G(x) - \frac{d-1}{d+2} J_W^R(x) \right] \quad (4)$$

where $x = M_n^2/M_S^2$. The functions $J_W^{G,R}(x)$, which arise from evaluation of the loop integrals in the case of graviton (G) and radion (R) respectively, are defined in Appendix B. In the above formula x_0 is an IR cutoff which is taken to be $x_0 = (R_c M_S)^{-2}$ following the suggestion of Ref. [10]. It corresponds to the mass of the first massive KK state of the graviton/radion. Using the relation [6] between R_c , M_S and d , it follows that

$$x_0 \simeq 2^{1/d} \times 10^{-62/d} \left(\frac{M_S}{1 \text{ TeV}} \right)^{4/d}, \quad (5)$$

which is clearly minute for $M_S \sim 1 \text{ TeV}$ and approaches unity when $M_S \rightarrow M_{Pl}^{(4)}$.

⁶such as, for example $\bar{f}fWW$ -type and $\bar{f}fZZ$ -type terms; there are many other possibilities.

Similarly, the final result for $\Pi_{ZZ}^{AADD}(0)$ takes the form

$$\Pi_{ZZ}^{AADD}(0) \simeq \frac{M_S^2}{720\pi} \frac{1}{(2\sqrt{\pi})^d} \frac{1}{\Gamma(d/2)} \int_{x_0}^1 dx x^{-1+d/2} \left[J_Z^G(x) - \frac{d-1}{d+2} J_Z^R(x) \right] \quad (6)$$

where, as before, the functions $J_Z^{G,R}(x)$ are defined in Appendix B. Substitution of Equations (4) and (6) in Equation (3) now yields the value of δT for a given d and value of M_S . Since the functions $J_{W,Z}^{G,R}(x)$ are extremely complicated (see Appendix B), the integrals in the above formulae are evaluated numerically.

Evaluation of the Feynman diagrams in Figure 1(c) and (d) is not directly possible because the Feynman rules are still not available. However, we can make an approximate estimate of the ‘seagull’-type graph using the Feynman rule given in Eqn. (84) of Ref. [10] for a pair of (spin-2) gravitons coupling to a pair of scalars. Assuming that the longitudinal components of the W and Z bosons are given by the Goldstone equivalence theorem, we can evaluate the ‘seagull’ graph. This adds to the function $J_{W,Z}^G(x)$ by a term which we denote $K_{W,Z}^G(x)$. The full form of $K_{W,Z}^G(x)$ is given in Appendix B. We then make the assumption that the two contributions due to the transverse modes are each equal to that due to the longitudinal mode. This corresponds to multiplying $K_{W,Z}^G(x)$ by a factor of 3. We next parametrize the error due to these approximations by introducing an unknown parameter ξ , so that the extra contribution reads

$$J_{W,Z}^G(x) \longrightarrow J_{W,Z}^G(x) + 3 \xi K_{W,Z}^G(x) \quad (7)$$

We next note that the diagrams in Figure 1(c) have the *same* particles running in the loop as the corresponding ones in Figure 1(d), and therefore, should be proportional to the same loop integrals. In fact, it will not matter which of the graviton states contributes to the loop integral, since the propagators are same, each having an identical tower of states. Of course, there will be explicit dependence on the graviton/radion in the vertex and in the propagator connecting the loop with the WW vertex. However, to a first approximation, we assume that all these can be lumped into a common contribution, also proportional to $K_{W,Z}^G(x)$. All the extra contributions due to the diagrams in Figure 1(c) are thus absorbed into the unknown parameter ξ . Any momentum dependence arising

from the vertices and propagators contributing to ξ can be removed by defining ξ to be an average value. We therefore, do not add any extra contribution to $J_{W,Z}^R(x)$.

While the above procedure does not sound very rigorous, it is the best that can be done until all the relevant Feynman rules become available. Calculation of these is a non-trivial task and it may be some time before they are available in the literature [21]. In principle, the value of ξ , which parametrizes our present state of ignorance of the effective theory to $\mathcal{O}(\kappa^2)$, can depend explicitly on M_S and on d . However, as we show in the next section, any such dependence is very weak, and therefore, we obtain an *á posteriori* justification for the approximation made here.

It is clear from the above that we have little or no *á priori* knowledge of what the parameter ξ should be, except that it should be of order unity (which follows from dimensional arguments). However, it is here that the IR divergences come to our help. We shall see in the following section that gravitonic contributions to δT are IR divergent for $d = 2, 3$ and 4. It has been argued above that these IR divergences are spurious and hence we demand that they should cancel out when we sum over all diagrams. Since $J_{W,Z}^{G,R}(x)$ and $K_{W,Z}^G(x)$ are individually IR divergent, we can *tune* the value of ξ to get cancellation of these divergences. We then claim that this is the value of ξ which would be obtained from a proper knowledge of the Feynman rules in this model⁷. The exact procedure is described in the following section.

4. Numerical Results and Discussion

Numerical evaluation of the integrals in Equations (4) and (6) is not completely straightforward. To see this, consider, for example, $M_S = 1$ TeV and $d = 2$, in which case, the IR cutoff comes out to be $x_0 \sim 10^{-31}$. The integral runs, therefore, over several orders of magnitude and, being IR divergent, receives its principal contribution from the smallest values of x_0 . It is, therefore, convenient to smoothen the integrand by the transformation $\xi = -\log x$, which allows the integration to be done using standard numerical

⁷This is in the same spirit as the determination of, for example, the three-gluon vertex in QCD from considerations of gauge invariance

techniques. For $d \leq 4$, the IR cutoff $y_0 = -\log x_0$ grows with M_S (corresponding to the increase in energy gap between the two lowest KK states), so that, for very large values of $M_S \sim M_{Pl}^{(4)}$ (which means $x_0 \simeq 1$) the range of integration gets pinched off. However, the pinch-off is very sharp and only takes place when $x_0 \simeq 1$ to great precision. Of course, such large values of M_S are uninteresting from the experimental point of view, but it is reassuring to note that even if we get unacceptably large values for the ρ -parameter at low values of M_S , there is always the possibility of having extra dimensions compactified to the Planck length.

In their leading order calculation of one-loop effects in the AADD model, the authors of Ref. [10] have shown that these corrections to the scalar mass can be written in terms of (dimensionless) loop integrals, of which two are

$$I_4 = \int_{x_0}^1 dx x^{-2+d/2}, \quad I_5 = \int_{x_0}^1 dx x^{-3+d/2}, \quad (8)$$

where x_0 is the IR cutoff described above. On evaluation these come out to be

$$I_4 = \frac{2}{d-2} \left(1 - x_0^{-1+d/2}\right), \quad I_5 = \frac{2}{d-4} \left(1 - x_0^{-2+d/2}\right). \quad (9)$$

Now, these are clearly IR divergent for $d < 2$ and $d < 4$ respectively. Even the use of the IR cutoff x_0 leads to extremely large values of these integrals, since it is clear that x_0 is an extremely small quantity (unless $M_S \rightarrow M_{Pl}^{(4)}$).

We now adopt the philosophy that the infrared divergences are spurious and must cancel out of a complete calculation. The main motivation for this — explained above — is that gravity is well-known to be infrared-safe, in fact, more so in dimensions greater than four, and this feature is expected to be preserved under compactification. The origin of the IR divergences may be attributed to a breakdown of the mass-continuum approximation in the low-momentum limit. Since we cannot make a complete analytic calculation and check the cancellation explicitly, the best we can do is to *tune* the value of the unknown parameter ξ and see if we can achieve cancellations in the different cases for d and M_S . Our results are shown in Figure 2, for $M_S = 1$ TeV, $M_H = 250$ GeV and $d = 2, 3$ and 4.

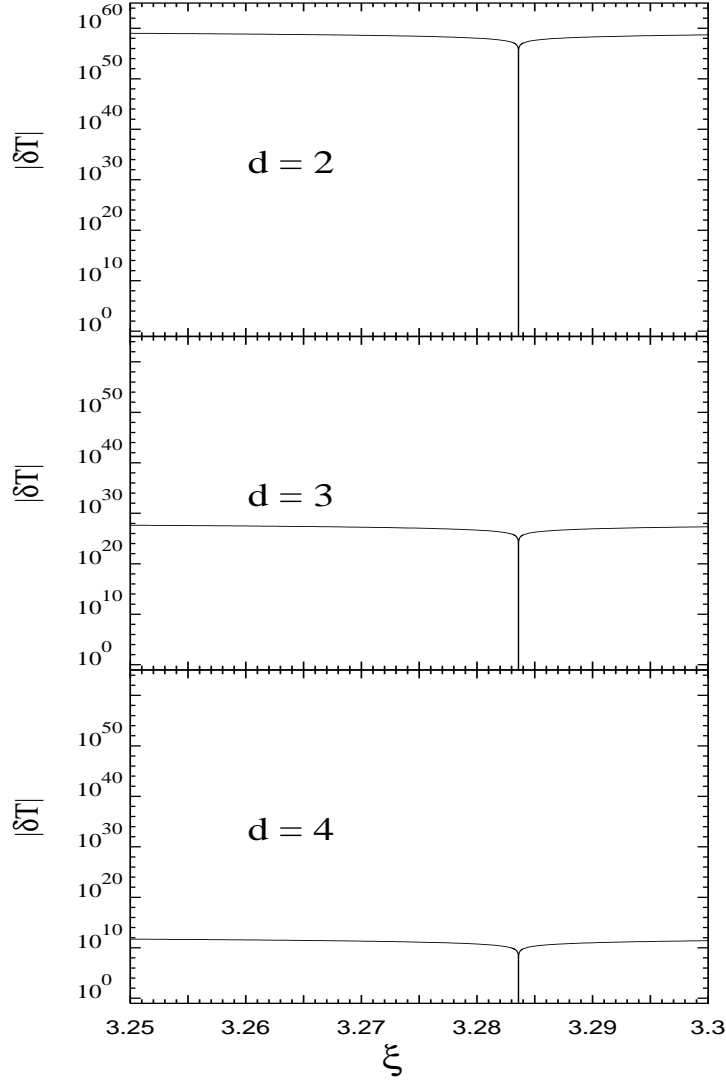


Figure 2. *Illustrating the cancellation of IR divergences in δT for $d = 2, 3$ and 4 . Note that these occur for the same value of ξ in each case. This is also insensitive to variations in M_S and M_H .*

In Figure 2, it may be seen that a sharp cancellation occurs for a specific value $\xi \simeq 3.2836$. Amazingly enough, we get *the same value* for all the three values of d . We also find that the value of ξ for which the cancellation occurs is almost completely insensitive to the value of M_S , while it has a very weak dependence on the mass of the Higgs boson (0.2% as M_H varies from 100 GeV to 700 GeV). This constant value of ξ may be attributed to the fact that it is almost entirely made up of constant or averaged factors in the unknown coupling, including κ, R_c and the gauge boson masses, apart from purely

numerical factors. We believe that a complete calculation, when available, will predict precisely this value $\xi \simeq 3.2836$. It is reassuring to note that the value of ξ is indeed of order unity as we expect.

Obviously, when such fine cancellations take place, it is not possible to predict the value of δT for $d = 2, 3$ and 4, since any result can be obtained by making small changes in ξ . However, in view of the fact that ξ is almost constant for $d = 2, 3$ and 4, we feel emboldened to extrapolate this value to the (IR-finite) cases $d = 5$ and 6 as well. We thus feed in this value of ξ for the cases $d = 5$ and 6, and are able to make concrete predictions for δT in the AADD model. Figure 3 shows our results for δT as a function of M_S (a) in the range of interest for collider experiments and (b) in the region where the UV-divergent integral gets pinched-off as $x_0 \rightarrow 1$. As before, we set $M_H = 250$ GeV, though there is little variation of the results with this parameter.

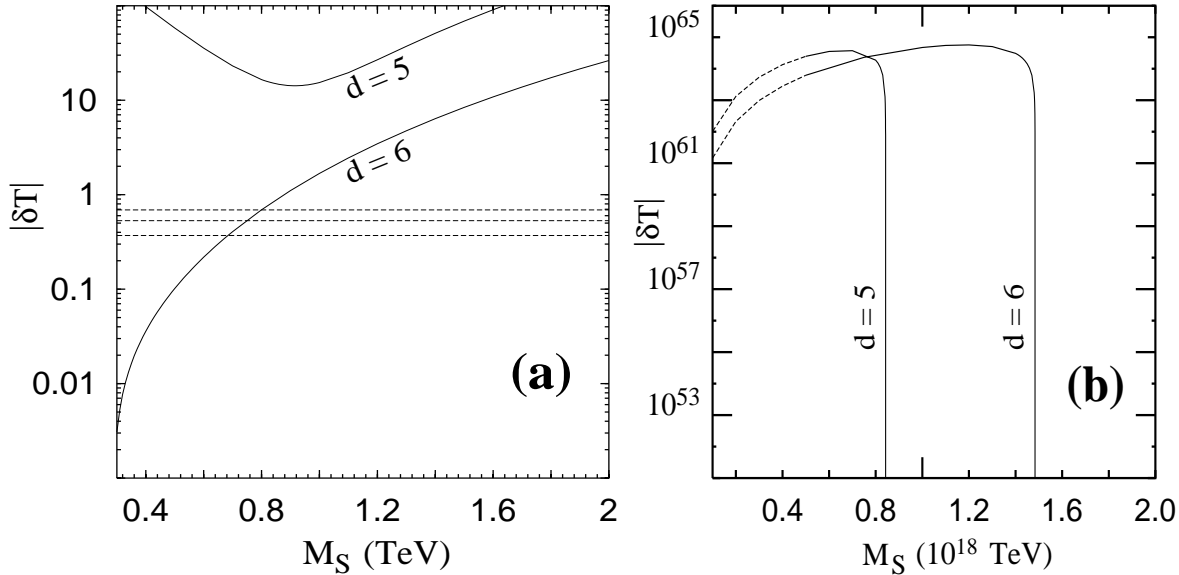


Figure 3. Variation in δT in the AADD model with increasing M_S , setting $\xi = 3.2836$. In (a), the dashed lines correspond (from bottom upwards) to the experimental upper limits at 1σ , 2σ , 3σ respectively. The pinching-off of the UV-divergent integral at the Planck scale is illustrated in (b).

As Figure 3 shows, the gravitational contribution to the T -parameter shows a non-decoupling behaviour, though, of course, it finally falls to zero when $M_S \rightarrow M_{Pl}^{(4)}$. The non-decoupling behaviour, we feel, is an indication that the theory is an effective one. In

fact, for $d = 5$, it appears that any value of M_S in the range displayed in Figure 3(a) is ruled out by experiment — which forces us to have $M_S \sim M_{Pl}^{(4)}$, as in Figure 3(b). For $d = 6$ we obtain an *upper* bound of about 725 GeV at 95% C.L., which is barely allowed by experimental data (see Table 1). A slight improvement in the data is likely to rule this out and this would force $M_S \sim M_{Pl}^{(4)}$ for $d = 6$ as well.

The actual scenario need not be so gloomy, however, since the effective theory of Refs. [9] and [10] is only an approximation and the underlying (string?) theory is believed to be finite. At the present juncture, it is not possible to make calculations in the exact theory, and it would, therefore, be necessary to construct a phenomenological extension of the effective theory, by adding extra (non-renormalizable) operators with unknown coefficients⁸ to the Lagrangian [22]. The coefficients may then be adjusted to produce acceptable values of the ρ (and T) parameter, even for $d = 5$ and 6. This is a messy business which smacks of fine-tuning and will not be attempted in this work.

Finally, it is worth noting that in all these calculations, the contribution due to the radion, which has much lower levels of divergence, both in the UV and IR regions, turns out to be negligible in comparison to that of the graviton. Its inclusion in all the above formulae is more for the sake of completeness than for numerical accuracy.

6. Conclusions

In this work, we have performed a careful study of the W and Z boson self-energy corrections using the effective theory of KK graviton (and radion) interactions developed in Ref. [10] from the original suggestion of Refs. [1, 4]. Since these are one-loop calculations, the results have several interesting features, which do not show up in tree-level calculations. The results turn out to be strongly divergent, both in the UV and IR regimes. Using a cutoff at either end, as suggested in Ref. [10], can lead to large values of the T -parameter (and hence of the ρ parameter). Knowing gravity to be IR-safe, we demand cancellation of the IR divergences and use this to determine the unknown ξ which

⁸This would be analogous to the twelve $\mathcal{O}(p^6)$ operators in chiral perturbation theory.

parametrizes our ignorance of the full set of Feynman rules in this model. Interestingly, we find that ξ is independent of M_S and the number of extra dimensions, and hence can be used to make concrete predictions for 5 and 6 extra dimensions. We then find that for $d = 5$, M_S is driven by the T -parameter constraint to the Planck scale, while for $d = 6$, a small window in M_S around $725 - 750$ GeV is still viable. We are unable to make any predictions for $d < 5$.

We wish to emphasize that while our results do seem to show that models with large extra dimensions do not work well for $d > 4$, all that this means is that the formalism of Ref. [10] does not work well for $d > 4$. A deeper (perhaps finite) theory should certainly provide better insights and more acceptable values of the ρ -parameter. It might also be possible to build a phenomenological theory by adding non-renormalizable operators to the effective Lagrangian, but that lies outside the scope of this work.

Note added: After this work was completed, we received the preprint in Ref. [25] where a similar calculation has been done, using a purely analytic approach. The rather striking difference in our results from theirs is probably due to the fact that tadpole diagrams, as in Fig. 1 (b) and (c) have not been taken into account in Ref. [25]. We thus believe that the calculation in Ref. [25], though elegant, is incomplete and should not be taken as evidence for a decoupling behaviour in M_S , at least for the ρ parameter.

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Appendix A: Feynman rules

The Feynman rules relevant for the calculation of the ρ parameter are as follows. The vertex for a pair of gauge bosons coupling to the graviton are given by an effective Lagrangian

$$\mathcal{L}_{eff} = i \left(T_{GW}^{\mu\nu\rho\sigma} G_{\mu\nu}^{(n)} + T_{RW}^{\rho\sigma} \Phi^{(n)} \right) \left[W_\rho^+ W_\sigma^- + \frac{1}{2} Z_\rho Z_\sigma + \frac{1}{2} A_\rho A_\sigma \right]. \quad (\text{A.1})$$

The graviton coupling $T_{GW}^{\mu\nu\rho\sigma}(p, q)$ is given by

$$T_{GW}^{\mu\nu\rho\sigma}(p, q) = -\kappa \left[(M_{W/Z}^2 + p \cdot q) C^{\mu\nu\rho\sigma} + D^{\mu\nu\rho\sigma}(p, q) \right], \quad (\text{A.2})$$

where $\kappa = \sqrt{16\pi G_N}$ and

$$C^{\mu\nu\rho\sigma} = \eta^{\mu\rho} \eta^{\nu\sigma} + \eta^{\mu\sigma} \eta^{\nu\rho} - \eta^{\mu\nu} \eta^{\rho\sigma}, \quad (\text{A.3})$$

$$\begin{aligned} D^{\mu\nu\rho\sigma}(p, q) &= \eta^{\mu\nu} p^\sigma q^\rho \\ &\quad - \left[(\eta^{\mu\sigma} p^\nu q^\rho + \eta^{\mu\rho} p^\sigma q^\nu - \eta^{\rho\sigma} p^\mu q^\nu) + (\mu \leftrightarrow \nu) \right]. \end{aligned} \quad (\text{A.4})$$

In the above equation, both W/Z momenta are assumed to be directed towards the vertex.

The radion coupling is given by

$$T_{RW}^{\rho\sigma} = \sqrt{\frac{8}{3(2+d)}} \kappa M_{W/Z}^2 \eta^{\rho\sigma}. \quad (\text{A.5})$$

The propagator for a graviton with momentum k is

$$\frac{i P_{\mu\nu\rho\sigma}(k)}{k^2 - M_n^2 + i\epsilon} \quad (\text{A.6})$$

where the polarisation sum $P_{\mu\nu\rho\sigma}$ is given by

$$\begin{aligned} P_{\mu\nu\rho\sigma}(k) &= + \frac{1}{2} \left(\eta_{\mu\rho} - \frac{k_\mu k_\rho}{M_n^2} \right) \left(\eta_{\nu\sigma} - \frac{k_\nu k_\sigma}{M_n^2} \right) \\ &\quad + \frac{1}{2} \left(\eta_{\mu\sigma} - \frac{k_\mu k_\sigma}{M_n^2} \right) \left(\eta_{\nu\rho} - \frac{k_\nu k_\rho}{M_n^2} \right) \\ &\quad - \frac{1}{3} \left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{M_n^2} \right) \left(\eta_{\rho\sigma} - \frac{k_\rho k_\sigma}{M_n^2} \right) \end{aligned} \quad (\text{A.7})$$

Finally, the propagator for a radion is given by

$$\frac{i(d-1)}{k^2 - M_n^2 + i\epsilon}, \quad (\text{A.8})$$

where the extra factor $(d-1)$ arises from the sum over different modes of the radion in each dimension.

In this Appendix we have closely followed the notation and conventions of Ref. [10].

Appendix B: Loop Integrals

The momentum integrals arising in one-loop calculations involving KK graviton and radion modes are calculated using an ultra-violet cutoff M_S , where M_S is the ‘string’ scale. We use the well-known notation of ’tHooft and Veltman and Passarino and Veltman [24] to describe momentum integrals of products of propagator functions. Then, the scalar one-point function turns out to be

$$\begin{aligned} A(m^2) &= \int \frac{d^4k}{\pi^2} \frac{1}{k^2 + m^2 + i\epsilon} \\ &= M_S^2 \tilde{A}\left(\frac{m^2}{M_S^2}\right) \end{aligned} \quad (\text{B.1})$$

where

$$\tilde{A}(x) = 1 - x \log\left(1 + \frac{1}{x}\right) \quad (\text{B.2})$$

As in Refs. [24], all integrals are defined in Euclidean space.

For the calculation of T , we require to calculate the boson propagators in the zero momentum limit. Thus, for the scalar two-point function, we define

$$\begin{aligned} \tilde{B}_0(x_1, x_2) = B_0(m_1^2, m_2^2; 0) &= \lim_{p^2 \rightarrow 0} \int \frac{d^4k}{\pi^2} \frac{1}{[k^2 + m_1^2] [(k+p)^2 + m_2^2]} \\ &= \frac{-1}{(x_1 - x_2)} \left[\tilde{A}(x_1) - \tilde{A}(x_2) \right] \end{aligned} \quad (\text{B.3})$$

where $x_{1,2} = m_{1,2}^2/M_S^2$. We are then in a position to define

$$\begin{aligned} B'_0(m_1^2, m_2^2; 0) &= \lim_{p^2 \rightarrow 0} \frac{\partial}{\partial p^2} B_0(m_1^2, m_2^2; p^2) \\ &= \frac{1}{M_S^2} \tilde{B}'_0(x_1, x_2) \end{aligned} \quad (\text{B.4})$$

which turns out, on evaluation, to be

$$\begin{aligned}\tilde{B}'_0(x_1, x_2) &= \frac{1}{x_1 x_2 (x_1 - x_2)^3} \\ &\times \left[x_1 - x_2 - x_1 x_2 \log\left(\frac{x_1}{x_2}\right) - (1 + x_1 x_2) \{x_1 \tilde{A}(x_2) - x_2 \tilde{A}(x_1)\} \right].\end{aligned}\quad (\text{B.5})$$

The vector two-point function is defined to be

$$\begin{aligned}B_\mu(m_1^2, m_2^2; p) &= \int \frac{d^4 k}{\pi^2} \frac{k_\mu}{[k^2 + m_1^2] [(k + p)^2 + m_2^2]} \\ &= p_\mu B_1(m_1^2, m_2^2; p^2),\end{aligned}\quad (\text{B.6})$$

which leads to

$$\begin{aligned}\tilde{B}_1(x_1, x_2) &= B_1(m_1^2, m_2^2; 0) \\ &= -\frac{1}{2} \left[\tilde{B}_0(x_1, x_2) + (x_1 - x_2) \tilde{B}'_0(x_1, x_2) \right].\end{aligned}\quad (\text{B.7})$$

The tensor two-point function is

$$\begin{aligned}B_{\mu\nu}(m_1^2, m_2^2; p) &= \int \frac{d^4 k}{\pi^2} \frac{k_\mu k_\nu}{[k^2 + m_1^2] [(k + p)^2 + m_2^2]} \\ &= p_\mu p_\nu B_{21}(m_1^2, m_2^2; p^2) + \delta_{\mu\nu} B_{22}(m_1^2, m_2^2; p^2),\end{aligned}\quad (\text{B.8})$$

and we define

$$B_{22}(m_1^2, m_2^2; 0) = M_S^2 \tilde{B}_{22}(x_1, x_2). \quad (\text{B.9})$$

This function can be evaluated as

$$\tilde{B}_{22}(x_1, x_2) = \frac{1}{6} \left[\tilde{A}(x_1) - 2x_2 \tilde{B}_0(x_1, x_2) + (x_1 - x_2) \tilde{B}_1(x_1, x_2) \right] \quad (\text{B.10})$$

A similar expression can be derived for B_{21} , but it is not relevant for the present calculation.

Now, in terms of these functions, we can write the integrands relevant for $\Pi_{WW}(0)$ as

$$\begin{aligned}J_G^W(x) &= I_G(X_W, x) \\ &- \frac{3X_W}{x} \left\{ 283 - 12 X_W \tilde{A}(X_W) - 6 X_Z \tilde{A}(X_Z) - 4 X_H \tilde{A}(X_H) \right. \\ &\quad \left. + 8 \sum_f C_f X_f \tilde{A}(X_f) \right\}\end{aligned}\quad (\text{B.11})$$

where $X_i = Mi^2/M_S^2$ and the colour factor $C_f = 3$ for quarks and 1 for leptons. Similarly

$$J_D^W(x) = 4X_W \left[16 I_D(X_W, x) - \frac{6}{x} \left\{ 295 + 24 X_W \tilde{A}(X_W) + 6 X_Z \tilde{A}(X_Z) + 2 X_H \tilde{A}(X_H) + 8 \sum_f C_f X_f \tilde{A}(X_f) \right\} \right] \quad (\text{B.12})$$

Similarly, we can write the integrands relevant for $\Pi_{ZZ}(0)$ as

$$J_G^Z(x) = \frac{1}{4} I_G(X_W, x) - \frac{3X_Z}{2x} \left\{ 283 - 12 X_W \tilde{A}(X_W) - 6 X_Z \tilde{A}(X_Z) - 4 X_H \tilde{A}(X_H) + 8 \sum_f C_f X_f \tilde{A}(X_f) \right\} \quad (\text{B.13})$$

$$J_D^Z(x) = 4X_Z \left[4 I_D(X_W, x) - \frac{3}{x} \left\{ 295 + 24 X_W \tilde{A}(X_W) + 6 X_Z \tilde{A}(X_Z) + 2 X_H \tilde{A}(X_H) + 8 \sum_f C_f X_f \tilde{A}(X_f) \right\} \right] \quad (\text{B.14})$$

The I -functions, which arise from evaluation of the diagram in Fig. 1(a), can be written as

$$I_D(x_1, x_2) = \frac{1}{2} \tilde{A}(x_1) + (3x_1 - x_2) \tilde{B}_0(x_1, x_2) + \frac{1}{2}(x_1 - x_2) \tilde{B}_1(x_1, x_2) \quad (\text{B.15})$$

for the loop integral involving radion exchange, and

$$I_G(x_1, x_2) = \frac{1}{x_1 - x_2} \frac{1}{x_2^2} \left[f_1(x_1, x_2) + f_2(x_1, x_2) \tilde{A}(x_1) + f_3(x_1, x_2) \tilde{A}(x_2) + f_4(x_1, x_2) \tilde{B}_{22}(x_1, x_2) \right] \quad (\text{B.16})$$

for the loop integral involving graviton exchange. The functions $f_i(x_1, x_2)$ are given by

$$f_1(x_1, x_2) = (x_1 - x_2) \left[12x_2^3 - 60x_1x_2^2 + 4(24x_1^2 - 19x_1 - 11)x_2 - (48x_1^3 + 98x_1^2 + 64x_1 + 33) \right] \quad (\text{B.17})$$

$$f_2(x_1, x_2) = 48x_1x_2 \left[12x_2^2 + x_1x_2 - 3x_1^2 \right] \quad (\text{B.18})$$

$$f_3(x_1, x_2) = 12 \left[x_2^2 - 47x_1^2x_2^2 + 10x_1^3x_2 - 4x_1^4 \right] \quad (\text{B.19})$$

$$f_4(x_1, x_2) = -48x_1(x_1 - x_2) \left[x_2^2 + 8x_1x_2 + x_1^2 \right] \quad (\text{B.20})$$

Finally, the ‘evaluation’ of seagull diagrams leads to the functions

$$\begin{aligned} K_W^G(x) &= 28.9 \frac{X_W}{x} \left[2 + 21x + 54x^2 \tilde{A}(x) \right] , \\ K_Z^G(x) &= 72.0 \frac{X_Z}{x} \left[2 + 21x + 54x^2 \tilde{A}(x) \right] . \end{aligned} \quad (\text{B.21})$$

References

- [1] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B429**, 263 (1998) . Some precursors of the model are V. Rubakov and M. Shaposhnikov, *Phys. Lett.* **B125**, 136 (1984) ; A. Barnaveli and O. Kancheli, *Sov. J. Nucl. Phys.* **51**, 573 (1990); I. Antoniadis, *Phys. Lett.* **B246**, 377 (1990) ; I. Antoniadis, C. Muñoz and M. Quiros, *Nucl. Phys.* **B397**, 515 (1993) ; I. Antoniadis, K. Benakli and M. Quiros, *Phys. Lett.* **B331**, 313 (1994) .
- [2] M.B. Green, J. Schwarz and E. Witten, *Superstring Theory*, Vols. 1 and 2 (Cambridge University Press, 1987).
- [3] For an excellent pedagogical review, see, for example, J. Polchinski, *Tasi Lectures on D-Branes*, hep-th/9611050 (1996).
- [4] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B463**, 257 (1998) .
- [5] J.C. Long, H.W. Chan and J.C. Price, *Nucl. Phys.* **B539**, 23 (1999) .
- [6] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Rev.* **D59**, 086004 (1999) .
- [7] N. Arkani-Hamed, S. Dimopoulos and J. March-Russell, SLAC preprint SLAC-PUB-7949 (1998), hep-th/9809124.
- [8] G. Nordström, *Phys. Zeitschr.* **15**, 504 (1914); Th. Kaluza, *Situngeber. Preuss. Akad. Wiss. Berlin*, p.966 (1921); O. Klein, *Z. Phys.* **37**, 895 (1926), 1938 Conference on New Theories in Physics, Kazimierz, Poland (1938) and *Helv. Phys. Acta*, Supp.**IV**, 58 (1956); For a fascinating discussion

of the early history of gauge and KK theories, see L. O’Raifeartaigh and N. Straumann, hep-ph/9810524; A more technical review is given by D. Bailin and A. Love, *Rept. Prog. Phys.* **50**, 1087 (1987).

- [9] G.F. Giudice, R. Rattazzi and J.D. Wells, *Nucl. Phys.* **B544**, 3 (1998) .
- [10] T. Han, J.D. Lykken and R.-J. Zhang, *Phys. Rev.* **D59**, 105006 (1999) .
- [11] R. Sundrum, *Phys. Rev.* **D59**, 085009 (1999) , *Phys. Rev.* **D59**, 085010 (1999) .
- [12] M. Graesser, LBL preprint LBNL-42812 (Feb 1999), hep-ph/9902310.
- [13] M. Veltman, *Act. Phys. Pol.* **B8**, 475 (1977).
- [14] T.G. Rizzo and J.D. Wells, SLAC preprint SLAC-PUB-8119 (Jun 1999), hep-ph/9906234; A. Strumia, Pisa U. preprint IFUP-TH-29-99 (Jun 1999), hep-ph/9906266; R. Casalbuoni *et al.*, U. of Florence preprint DFF-341/7/99 (Jul 1999), hep-ph/9907355.
- [15] E.A. Mirabelli, M. Perelstein and M.E. Peskin, *Phys. Rev. Lett.* **82**, 2236 (1999) ; K. Cheung and W.-Y. Keung, Davis preprint UCD-HEP-99-6 (1999), hep-ph/9903294; K. Cheung, Davis preprint UCD-HEP-99-8 (1999), hep-ph/9904266; D. Atwood, S. Bar-Shalom and A. Soni, Brookhaven preprint BNL-HET-99/11, hep-ph/9903538; S. Cullen and M. Perelstein, *Phys. Rev. Lett.* **83**, 268 (1999) ; T. Han, D. Rainwater and D. Zeppenfeld, Madison MADPH-99-1115 (May 1999), hep-ph/9905423; D.K. Ghosh, P. Poulose and K. Sridhar, TIFR preprint TIFR-TH-99-47 (1999).
- [16] S. Cullen and M. Perelstein in Ref. [15], hep-ph/9903422;
- [17] T.G. Rizzo, *Phys. Rev.* **D59**, 115010 (1999) ; X.-G. He, Taiwan preprint (1999), hep-ph/9905295; K. Agashe and N.G. Deshpande, *Phys. Lett.* **B456**, 60 (1999) ; T.G. Rizzo, SLAC preprint SLAC-PUB-8114 (1999), hep-ph/9904380; J.L. Hewett, *Phys. Rev. Lett.* **82**, 4765 (1999) ; P. Mathews, S. Raychaudhuri and K. Sridhar, TIFR preprint TIFR-TH/99-13 (1999), hep-ph/9904232; P. Mathews, S. Raychaudhuri and K. Sridhar, *Phys. Lett.* **B450**, 343 (1999) ; T.G. Rizzo, SLAC preprint SLAC-PUB-7986 (1999), hep-ph/9902273; K.Y. Lee, *et al.*, Seoul preprint SNUTP-99-022 (1999), hep-ph/9905227. P. Mathews, S. Raychaudhuri and K. Sridhar, *Phys. Lett.* **B455**, 115 (1999) ; C. Balasz, *et al.*, Michigan State U. preprint MSUHEP-90105 (1999), hep-ph/9904220; K.Y. Lee, H.S. Song and J.H. Song, Seoul preprint SNUTP-99-021 (1999), hep-ph/9904355; T.G. Rizzo, SLAC preprint SLAC-PUB-8071 (1999), hep-ph/9903475; D. Atwood, S. Bar-Shalom and A. Soni, Riverside preprint UCRHEP-T258 (Jun 1999), hep-ph/9906400; X.-G. He, hep-ph/9905295 P. Mathews,

- P. Poulose and K. Sridhar, *Phys. Lett.* **B461**, 196 (1999) ; D.K. Ghosh, P. Mathews, P. Poulose and K. Sridhar, *JHEP* **9911**, 004 (1999).
- [18] A. Gupta, N.K. Mondal and S. Raychaudhuri, TIFR preprint TIFR-HECR-99-02 (Apr 1999), hep-ph/9904234.
- [19] P. Mathews, S. Raychaudhuri and K. Sridhar in Ref. [17], hep-ph/9904232. .
- [20] M. Peskin and T. Takeuchi, *Phys. Rev. Lett.* **65**, 964 (1990) and *Phys. Rev.* **D46**, 381 (1992) .
- [21] P. Das, P. Jain, S. Panda and S. Raychaudhuri, *work in progress*.
- [22] K. Huitu and T. Kobayashi, Helsinki preprint HIP-1999-40-TH (Jun 1999), hep-ph/9906431.
- [23] Review of Particle Physics, *Eur. Phys. J.* **C3**, 1 (1998) .
- [24] G. 'tHooft and M. Veltman, *Nucl. Phys.* **B153**, 365 (1979) ; G. Passarino and M. Veltman, *Nucl. Phys.* **B160**, 151 (1979) .
- [25] J.R. Espinosa and R.-J. Zhang, Madrid preprint FT-UAM-CSIC-00-09 (March 2000), hep-ph/0003246.